[No 32.]

G.W.R. Mechanics' Institution.

JUNIOR ENGINEERING SOCIETY.

TRANSACTIONS, 1899-00

ORDINARY MEETING.– Monday, December 11th, 1899.
Chairman – Mr. J. W. Cross, Stud.Inst.C.E.

"LOCOMOTIVE BALANCING,"

BY

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The best introduction to the subject of balancing would appear to be the consideration of the simplest of all cases in which balancing is necessary. This is clearly the case of a revolving shaft with a weight fixed at the end of an arm, which is attached to the shaft at some point.

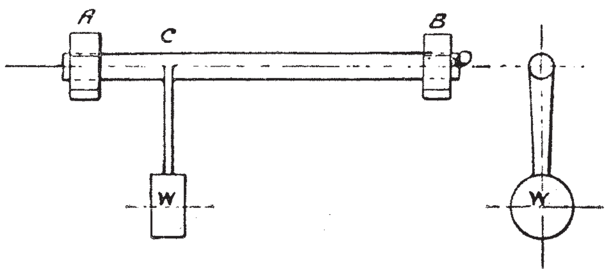

Fig. 1.

Fig. 1 shows such a shaft, which has a weight, W, attached to an arm fixed at the point C. Now, suppose the shaft, together with the weight W, to rotate. At once centrifugal action comes into play, producing a force pulling at C in the direction of the length of the arm ; that is to say, along C W. Anyone may satisfy himself that this force exists by swinging a weight round on the end of a piece of string. The reason why the force exists is almost as simple as the practical proof of its existence.

One of Newton's laws of motion states that any moving body tends to continue moving at the same pace, and also to move in a straight line. Whenever a heavy substance of any kind moves along a curved path, there must be some force applied to cause it to do so. Also, as long as the path remains curved, the force must continue to act. Now, the weight at the end of the arm follows a circular path, which is obviously curved at all points.

Therefore, there must always be a force acting on it to pull it, so to speak, into this circular path. This force is supplied by a tension in the arm C W. This tension produces a pull on the weight W, which overcomes the centrifugal force due to rotation, and at the same time produces a pull on the shaft at C ; and the method of balancing this pull it is the object of this paper to explain. It will probably suggest

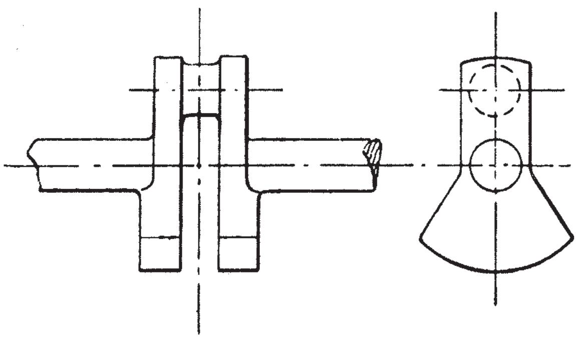

Fig. 2

itself to everyone that the simplest way to balance this force would be to place an equal weight on an arm of equal length exactly opposite to the weight W. This is a perfectly true and ideal method, but unfortunately there are insurmountable difficulties in the way of its practical application.

The nearest possible approach to it consists in extending the webs on the opposite side of the crank to the crank pin, and there placing balance weights as in Fig. 2. This method is sometimes seen in gas engines and high speed steam engines. In the case of locomotives, however, the weights to be balanced would require the balance weights to be excessively massive and bulky, or else cause the length of the webs to be so great as to be inadmissible in the case of inside cylinder engines with any reasonable height of boiler centre. The expense would also be greater than with the present form of balancing. In locomotives, the almost universal practice is to place balance weights on the wheels just inside the tyres. The equivalent, to this in the simple case now under consideration will be to place weights on arms attached to the ends of the shaft. The function of these weights will be to produce such centrifugal forces as will place the shaft in a state of equilibrium. The problem of finding these forces is identical with that of finding the reactions of a beam bearing an isolated load. Fig. 3 represents the shaft A B ; at C is the force F produced by the rotating weight. The forces f_1 and f_2 at A and B respectively are =

$$(1)\ \frac{F}{1 + \frac{x}{y}} \quad \text{and} \quad (2)\ \frac{F}{1 + \frac{y}{x}}$$

and act in the opposite direction to F. This is equivalent to asserting

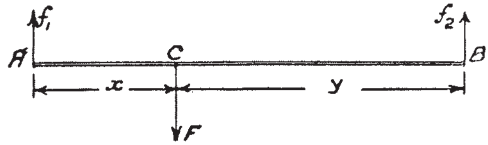

Fig. 3.

that the balance weights must be exactly opposite to the weight to be balanced when the shaft is viewed from either end.

The next problem is : Given the magnitude of the centrifugal forces f_1 and f_2 to find the weights which are necessary to produce them, and at what distance from the centre these weights must be placed. This involves the use of the formula for finding centrifugal force (3) F= 1·24 r N² W where F = centrifugal force, r = distance of centre of gravity of rotating weight from axis of rotation in feet, N = number of revolutions per second, and W = weight of rotating mass. Now call the weight to be balanced W at radius R, and let the centrifugal force it produces be F. In the same way let the balance weights be w_1 and w_2, their radii r_1 and r_2, and the centrifugal forces they produce f_1 and f_2. The number of revolutions will be N in both cases, as all rotate together. Then (4) $F = 1{\cdot}24\ RN^2\ W$. (5) $f_1 = 1{\cdot}24\ r_1\ N2\ w_1$ and (6) $f_2 = 1{\cdot}24\ r\ N^2\ w_2$.

It was shown earlier in the paper, in equations 1 and 2, that

$$f_1 = \frac{F}{1 + \frac{x}{y}} \text{ and } f_2 = \frac{F}{1 + \frac{y}{x}}$$

where x and y are the distances A C and C B respectively in Fig. 3. Substituting the values just obtained for F_1 f_1 and f_2, the equations 1 and 2 become equations (7) and (8), and dividing both sides of these equations by $1 \cdot 24\ N^2$, equations (9) and (10) are obtained.

Now, R_1 W_1 x and y are all known quantities, and therefore r_1 w_1 and r_2 w_2 can be found. That is to say, the product of the balance weight, by the distance from the centre of rotation, is known for each end of the shaft. It follows that, choosing any weight and dividing this product by it, the distance the weight must be placed from the centre of the shaft is determined ; or choosing the distance from the centre, the required weight may be found. In all cases where the distance of a weight from any point is referred to, the distance of the centre of gravity of the weight is implied.

It is worth noting that adding equations (9) and (10) together, equation (11) is obtained. Now, supposing the shaft to be rotated till all the arms stand horizontal and to retain them in this position, r_1 w_1 becomes the moment of the weight w_1 round the centre of the shaft, r_2 w_2 the moment of w_2 and R W the moment of W, and equation (11) proves that the sum of the moments of the balance weights is equal to the moment of the weight to be balanced. This being so, the shaft will have no tendency to rotate however the arms may stand. The elimination of any tendency to rotate does not, however, necessarily create a perfect balance unless equations (9) and (10) are also satisfied. This is only one of an infinite number of methods of satisfying equation (11). As an example of the manner in which equation (11) may be assured without creating the balance, let $w_2 = O$, and in this case $r_2\ w_2 = O$, and therefore $r_1\ w_1 = R\ W$. That is to say, the whole of the balancing is done at one end of the shaft. The effect of this is shown in Fig. 4. The balance weight produces the force f_1 at A, the weight to be balanced produces the force F at C. Clearly, when these forces act as shown in the figure, there is an upward pressure on the bearing at A and a downward pressure at B. After half a revolution, however, the forces are exactly reversed, giving a *downward* pressure at A and an upward pressure at B. This is evidently not the result aimed at in making a balance. The next point for consideration is the effect of the connecting rod, piston rod, etc., on the balance of an ordinary stationary engine. It must be remembered that a stationary engine is, as a rule, fixed to solid foundations and is often a very slow running engine. Many stationary engines are, therefore, balanced by a single weight on the flywheel only, or not at all.

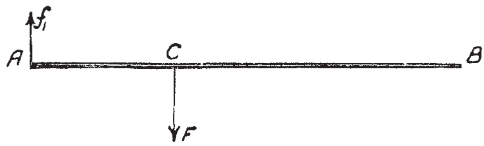

Fig. 4.

Most stationary engines balanced on scientific principles are high speed engines, and these are seldom balanced by any other method than that of placing weights on extended crank webs. A locomotive engine is a high speed engine, often running between three and four hundred revolutions per minute, and is also perfectly suited to show up any defects in balancing, being mounted on springs and free to move longitudinally. A single cylinder stationary engine is, however, more suitable than the complicated locomotive for explaining the principle of balancing. Fig. 5 represents diagramatically the crank, connecting rod, piston rod, etc.,

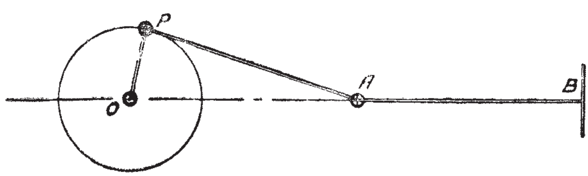

Fig 5

of such an engine. O is the centre of the crank shaft, and O P the crank arm rotating round O. Clearly the end of the connecting rod near P, including all the weights connected with it, such as big end brasses, straps and cottars, for all practical purposes, rotate with P, and in so doing, produce centrifugal forces, acting through the crank webs, to the crank shaft. It is equally clear that the end A does not rotate at all, and, therefore, does not produce any centrifugal forces. All the intermediate parts of the connecting rod move in oval paths, and produce, more or less, the effect of rotating weights according to their proximity to P or A.

From this it follows that a certain portion only of the connecting rod must be considered as a rotating weight moving round O at the same radius as the crank pin. This portion of the weight may be approximately equal to the weight of the big end plus half the weight of the body of the rod. There are numerous formulæ, some simple, others extremely complicated, for obtaining the portion of the connecting rod which ought theoretically to be considered as rotating. Some of these formulæ are exact, some only approximations. The subject does not, however, appear to be one of vital importance, for the following reasons : The simple formula already mentioned will give results with an error of probably not more than one-fifteenth or one-twentieth of the whole weight of the rod. Also the question is not one of balancing or not balancing, but only one of balancing the whole or two-thirds of the weight, as that portion of the rod not treated as rotating is treated as a reciprocating weight, and as such about two-thirds of it is balanced. The total possible error is, therefore, only about one-forty-fifth or one-sixtieth of the weight of the connecting rod. A little consideration of some of the assumptions usually made when calculating balance weights will show that this error is altogether insignificant compared with others that are unavoidable.

The effect of the reciprocating parts must now be considered. These consist of the crosshead, piston rod, piston head, and that portion of the connecting rod not taken as rotating. Anyone standing with a pair of heavy dumb-bells, and striking out quickly forwards several times, will soon be convinced that reciprocating weights require some attention. The body will be found to move back as the arms and dumb-bells move forward, and *vice versa*. This is somewhat the same effect as that produced by the recoil of a gun : the shot moves forward and the gun moves backward.

The same things happens in an engine. When the piston, etc., move in one direction—or, to be more accurate, alter their rate of motion—the engine tends to move bodily in the other direction. In the case of an engine bolted to a firm foundation and running slowly this is of little importance, but in a locomotive mounted on wheels, and therefore free to move longitudinally, it is of the greatest importance to balance these reciprocating weights as perfectly as possible, otherwise the disturbing forces produced will cause both plunging and oscillation about a vertical axis. The only perfect way of overcoming these disturbances would be to cause other reciprocating masses to move in exactly the opposite direction to the piston, etc., at all points of the stroke. This form of balancing has been employed in an American four-cylinder locomotive. A good example of the principle of this form of balancing is the swing of the arms of a person walking. As the right leg goes forward the right arm goes backwards, and *vice versa*. The body is kept comparatively steady, as may easily be noticed if a few experiments be made. The difficulty of balancing these moving parts of an engine by balance weights attached to the wheels in the usual manner is as follows :—The acceleration of any rotating mass is towards the centre of rotation. This is deduced directly from the fact already mentioned that the force over-

Fig. 6

coming centrifugal action acts towards the centre of rotation. It follows, therefore, that when the crank connecting rod and piston are all in a straight line, as in Fig. 6, since P is being accelerated towards O, the acceleration of the whole connecting rod and also the piston rod, etc., must be approximately the same as that of P, and therefore the balance weight must be as large as though all these weights were actually rotating with P. However, when the crank has turned to the position in which it is shown in Fig. 5, the acceleration of the piston rod, etc., would be practically zero, as the acceleration of P is at right angles to the length of the connecting rod. The speed at which the rod is turning round A is therefore only affected, and the speed with which A is moving cannot possibly be altered. At this point, therefore, the balance weight required for the reciprocating parts is zero ; hence the difficulty.

If balance weights are not provided, the reciprocating parts will be unbalanced when the crank is horizontal, and will produce a longitudinal plunging tendency, whilst giving no trouble when the crank is vertical, and does not require balancing.

If a full balance is used, this plunging tendency will be destroyed when the crank is horizontal, but the balance weight will be left free to create vertical disturbances when the crank is vertical.

In practice, a portion of the reciprocating parts are balanced, thus cutting down both the vertical and horizontal disturbances till they are of a magnitude which will not affect the safe running of the locomotive, or damage the permanent way. This portion of the reciprocating weights is usually about two-thirds of the weight of the true reciprocating parts, together with two-thirds of the weight of that portion of the connecting rod not treated as rotating.

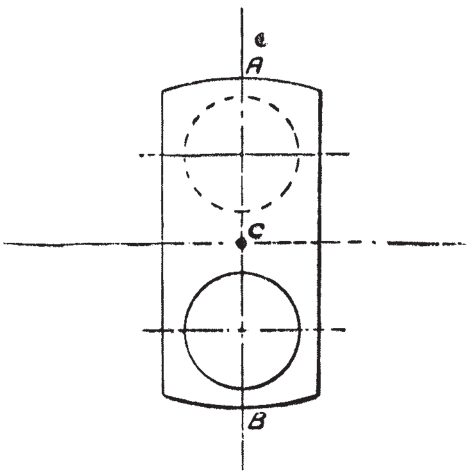

Fig 7

It now remains to deal more particularly with balancing as applied to locomotives. Firstly, it will be well to note the essential differences between the simple engine already considered and the locomotive. To begin with, the true rotating weights are not all concentrated at the crank pin, as assumed in the diagrammatic engine ; there is the weight of the crank webs to be taken into account. Secondly, there are two cranks at right angles to each other. Thirdly, in coupled locomotives there is the balance of the coupling rods to be considered, and also the distribution of the balance of the reciprocating parts between the pairs of wheels. These points will be explained as far as possible separately and in order. Beginning with the weights that are not concentric with the crank pin, and whose centre of gravity does not lie, therefore, on the centre line of the crank pin, it is obvious, on referring to Fig. 7, that all these weights are symmetrical about the line joining the centres of the crank pin and shaft. The parts to be considered consist of the webs shown evenly distributed on both sides of A B. Their centre of gravity will, therefore, lie on A B. The exact position can be determined by ordinary rules ; in this case it is at point C. Referring to the formula already given for centrifugal force, it will be seen that this force varies directly as the weight, and also directly as the radius at which this weight rotates. It is therefore justifiable to take a weight less than the weight of the webs as acting at the crank pin in the place of the actual weight of the webs acting at C. By simple proportion this reduced weight is found to be equal to the weight of the webs, multiplied by the distance

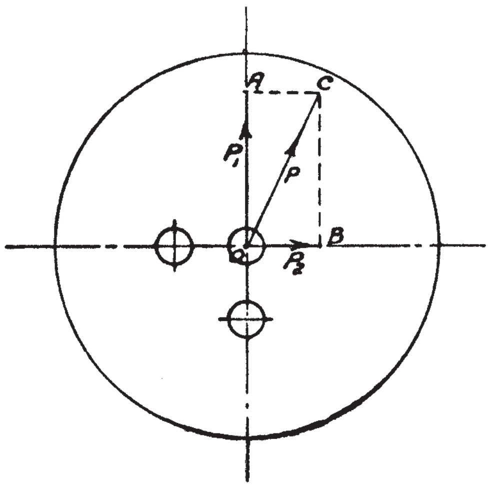

Fig. 8.

of their centre of gravity from the centre of the shaft, and divided by the radius of the crank pin. The weight so obtained is then added to the other weights acting at the crank pin.

Probably the simplest method of calculating the two cranks of a locomotive is to treat them separately. This will give for each wheel the value of two products, each of a weight multiplied by a radius. The forces produced by the two balance weights which would thus be obtained for each wheel are known to act at right angles, as they each act exactly opposite to the crank they are balancing, and these cranks are themselves at right angles. Fig. 8 represents a driving wheel ; the two forces may be called P_1 and P_2. The two balance weights would produce these forces if placed on the wheel. Any two forces, however, acting at one point will have exactly the same effect as a certain resultant force determined by drawing a parallelogram of forces. Thus, if O A represents P_1 to scale, and O B represents P_2, the diagonal O C of the completed parallelogram O A, C B will represent the resultant force P to the same scale. Now to find the product of the required single balance weight and its radius of action, this force P must be divided by $\cdot 31\ N^2$. If, however, instead of taking O A and O B to represent the forces produced by the two separate balance weights, they had been taken as representing the products of these balance weights into their respective radii, then the line O C will represent the product of the required single balance weight into the distance of its centre of gravity from the centre of the wheel.

Having obtained this product, the actual balance weight may easily be found on the principle of trial and error. That is to say, an assumption is made at the probable distance of the centre of gravity of the balance weight from the centre of the wheel. The product is then divided by this radius and the balance weight obtained. If it is now found that the centre of gravity of the weight lies a little nearer to the centre of the wheel than was assumed in the first instance, a second attempt must be made, taking a smaller radius, and so on. One or two attempts will certainly give the required weight and radius. The centre of gravity of the weight is most simply found by means of a cardboard or thick paper template, a plumb line and a needle. In calculating the weight of the balance, allowance must be made for those portions of the spokes falling within the balance weight, as they take no part in the balance, being themselves balanced by the corresponding spokes on the other side of the wheel.

The next point to be considered is the influence of coupling rods. All the parts of these rods move in circular paths, and therefore produce equal centrifugal forces at all points in their paths. These forces always act in a direction parallel to the centre line of the outside cranks, and are transmitted to the crank pin by the coupling rods themselves. They produce various stresses in the coupling rods : a bending stress when the cranks are vertical, and tension or compression when the cranks are horizontal. They may therefore be taken as acting equally at both of the outside crank pins. That is to say, half the weight of the coupling rod may be taken as concentrated at each crank pin working in it. In the case of single-framed engines, where the crank pins are fastened directly to bosses on the wheels, each coupling rod may be taken as acting on its own wheel only without affecting the other. In this case a balance weight may be calculated as exactly opposite the crank pin, on the principle of moments, and afterwards combined with the weight balancing the cranks, etc., by the parallelogram of forces as before mentioned. In the case of double-framed

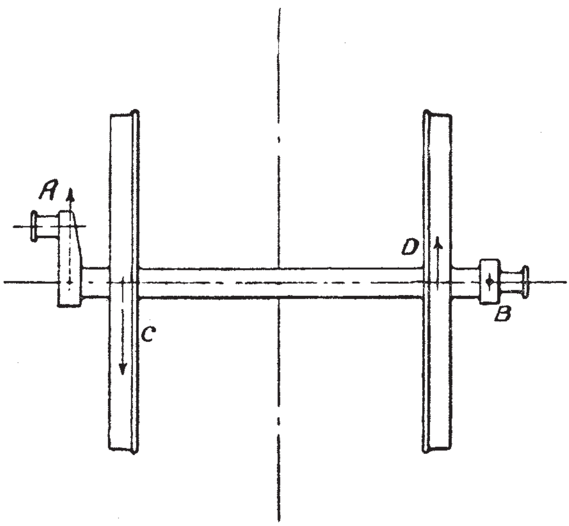

Fig. 9

engines, where the centrifugal force from the coupling rod acts at some distance from the wheel, it must be treated in the same manner as the forces from the cranks, and worked out in the same way as the reaction of a loaded overhanging beam. Fig. 9 will illustrate this.

If A B is the whole length of the crank shaft, at A the force produced by the coupling rod will be exerted, at C there must be a slightly larger force opposite to that at A, and at D a very small force on the same side as the force at A. These forces at D and C must be produced by balance weights on the wheels. These balance weights will then be combined with the larger balance weights as before. Given the magnitude of the force at A, and the distances of C and D from A, the forces at these points are easily determined as follows :—

$$\text{The force at D} = \frac{\text{force at A} \times \text{AC}}{\text{AD}}$$

and the force at C = the force at A + the force at D.

The weight of the eccentrics and their straps and rods may as a rule be neglected, as the radius at which they rotate is very small. If, however, it is desired to balance the centrifugal forces produced by them, it may be done on exactly the same principle as the other forces acting on the crank shaft. They may be treated separately to find the required balance weights or products for each wheel, and these balance weights or products may afterwards be combined with others on the same wheel by the parallelogram of force.

The only remaining point to be considered is the distribution of the balance of the reciprocating parts between the various pairs of wheels of a coupled engine. This is, however, an important point, and cannot by any means be ignored.

It has already been shown that the balance weights provided on account of the reciprocating masses have no forces to balance when they are vertically above or below the centre of the axle. This is caused by the reciprocating masses moving only in a horizontal plane, and therefore producing no vertical disturbances. It therefore follows that the centrifugal forces produced by the balance weights when in these positions act on the crank shaft, but have no forces to balance them. Now on each pair of wheels there is a certain load, transmitted to them through the springs and axle boxes. This load is fairly constant, as the springs allow for inequalities in the rails to a great extent. Without any disturbing forces, therefore, the pressure between the wheels and the rails would be fairly constant. The effect of the weights used to balance reciprocating parts has been shown to produce a variation in this otherwise constant pressure, as it first adds and then subtracts the centrifugal force produced by the balance weight. It can easily be seen that for two reasons a limit must be fixed to this disturbance.

Firstly, the pressure produced between the wheels and the rail when the balance weight is at its lowest point, and the centrifugal force acts downwards, must not be larger than the maximum load allowed for each wheel by those in charge of the permanent way.

Secondly, when the weight is at its highest point, the centrifugal force must not be sufficient to overcome the downward force due to the portion of the weight of the engine carried by the wheel, and therefore to lift the wheel clear off the rail. The weight allowed on the permanent way necessarily varies considerably with rails of different weights. To ensure that the wheel remains in contact with the rail, the maximum centrifugal force must not be more than a certain fraction of the static load on the wheel. The fraction is of course an arbitrary one, and depends to a great extent on the nature of the road.

Having obtained the maximum centrifugal force that may be allowed to act on each wheel, it is a very simple problem to calculate the balance weight to produce this force at the maximum speed the locomotive under consideration is likely to run. This, therefore, fixes the maximum allowable weight for the driving wheels as far as reciprocating masses are concerned. The balance weight for rotating masses may be as large as required, as it produces no vertical disturbing forces. It is often found, therefore, that it is impossible to place all the balance for reciprocating parts in the driving wheels. The remainder of the balancing has then to be divided between the other pairs of coupled wheels. The forces produced then act through the spokes, axle boxes and horns on to the frames, so steadying the engine.

These balance weights are combined with those required by the coupling rods and their cranks by the parallelogram of forces, as in previous cases where two weights have been found for one wheel. In some cases, even where the whole of the reciprocating masses might be balanced on the driving wheel, without producing any excessive pressure on the rails, or tendency to rise, this is not done, as it is considered better to distribute the weights between the various pairs of wheels.

There is no absolute reason why the reciprocating parts should not be balanced wholly on the leading or trailing wheels so long as these wheels are coupled to the driving wheels. There would appear to be some arguments in favour of dividing the balance between the leading, driving and trailing wheels (supposing the locomotive under consideration to be six coupled), so that the difference between the maximum centrifugal force and the portion of the weight of the engine carried by the wheel may be the same in all three. There would then be an equal margin allowed against lifting in each pair of wheels. If, however, it is desired to reduce the load on the rails, obviously the pair of wheels carrying least of the deadweight of the locomotive should take the maximum portion of the balance of reciprocating parts so long as this will not cause the wheel to lift. Probably this question may be best settled by actual experiments, as so many other considerations affect the steady running of various classes of engines, that no hard and fast rule may be laid down to suit them all.

The impossibility of satisfactorily balancing reciprocating parts is an excellent reason for making these parts as light as possible, apart from the enormous and unnecessary stresses set up by every pound of excessive weight in the piston, piston rod and crosshead.

In conclusion, it may be noted that it is useless to make calculations affecting balance weights with any great accuracy, as the assumptions made throughout the theory of balancing are to a great extent approximate. Also the reciprocating masses can not in any case be perfectly balanced ; the choice of the fraction of these weights to be considered is an arbitrary one, and leads to far greater divergency in the final results than could possibly be produced by neglecting decimals throughout the calculations.

LIST OF EQUATIONS.

1. $f_1 = \dfrac{F}{1 + \dfrac{x}{y}}$

2. $f_2 = \dfrac{F}{1 + \dfrac{y}{x}}$

3. $F = 1{\cdot}24\ r\ N^2\ W$

4. $F = 1{\cdot}24\ RN^2\ W$

5. $f_1 = 1{\cdot}24\ r_1\ N^2\ w_1$

6. $f_2 = 1{\cdot}24\ r_2\ N^2\ w_2$

7. $1{\cdot}24\ r_1\ N^2\ w_1 = \dfrac{1{\cdot}24\ RN^2\ W}{1 + \dfrac{x}{y}}$

8. $1{\cdot}24\ r_2\ N^2\ w_2 = \dfrac{1{\cdot}24\ RN^2\ W}{1 + \dfrac{y}{x}}$

9. $r_1\ w_1 = \dfrac{RW}{1 + \dfrac{x}{y}}$

10. $r_2\ w_2 = \dfrac{RW}{1 + \dfrac{y}{x}}$

11. $r_1\ w_1 + r_2\ w_2 = \dfrac{RW}{1 + \dfrac{x}{y}} + \dfrac{RW}{1 + \dfrac{y}{x}} = RW$

r = dist. of C of G of rotating mass from axis of rotation in feet

N = number of revolutions per sec

W = weight of rotating mass.